Unit 2: Polynomials

Student Edition

Unit 2.1 Interpret the Structure of Expressions

Student Learning Targets (SWBAT):

- Interpret parts of expressions such as terms, factors, coefficients
- Use the structure of an expression to identify ways to rewrite it
- Add polynomials
- Subtract polynomials
- Multiply polynomials

Notes:

Assignment 2.1:

Determine which are polynomial functions. State the degree, leading coefficient. For those that are not functions explain why.

1. $f(x) = 3x^{-2} + 17$

2. $f(x) = 2x^5 - \frac{1}{2}x + 9$

3. $f(x) = -9 + 2x$

4. $h(x) = 13$

5. $k(x) = \sqrt[3]{27x + 8x^3}$

Add or subtract the following polynomials

6. $(x^4 + 8 - 5x^3) + (2x^3 - 2 - 3x^4)$

7. $(4a^2 - 2 - 8a^3) + (5a^2 - 4a + 8a^3)$

8. $(6x^2 + 7x + 3x^3) - (5x^2 + 3x + 7x^3)$

9. $(8x^3 - 4x^2 + 5x^4) - (5x^3 - 5x^4 + 6x^2)$

10. $(2 - 3i) + (6 + 5i)$

11. $\left(\sqrt{5} - 3i\right) + (-2 + \sqrt{-9})$

12. $(2 - 3i) + (3 - 4i)$

13. $(2 - i) + (3 - \sqrt{-3})$

Find each product

10. $(5m-6)(2m+7)$

11. $(7r+8)(3r-5)$

12. $(2x-4)^2$

13. $(\sqrt{-4}+i)(6-5i)$

14. $(3n^3+2n)(4n^2-1)$

15. $(6x^3+2x)(x+5)$

16. $(2x+3)^3$

17. $(2+3i)(2-i)$

18. $(2-i)(1+3i)$

19. $(7i-3)(2+6i)$

Find the area and perimeter of the rectangles.

21.

$(3x^2-2x+1)$

$(x-3)$

22.

$(3x^2+4x-1)$

$(2x-4)$

2.2: Factoring Polynomials

Objectives:

- Factor expressions with a common factor
- Factor expressions when a = 1
- Factor expressions when a > 1

Notes:

Assignment 2.2

Factor each expression completely.

1. $3x^2 + 10x + 3$
2. $3x^2 - 13x + 12$
3. $9x^3 + 3x^2 - 12x$
4. $2x^3 - x^2 - 3x$
5. $14u^2 - 33u - 5$
6. $3x^4 + 7x^3 + 2x^2$
7. $x^2 + 9x + 14$
8. $y^2 - 11y + 30$
9. $6t^2 + 5t + 1$
10. $12x^2 + 11x - 15$

Solve each equation using factoring.

11. $x^2 + 9x - 22 = 0$
12. $3x^2 + 8x + 4 = 0$
13. $x^4 - 29x^2 + 100 = 0$
14. $4x^4 - 14x^2 + 12 = 0$
15. $x^6 - 9x^4 - x^2 + 9 = 0$
16. $4x^4 - 4x^2 - x^2 + 1 = 0$

Unit 2.3 – Factoring Special Polynomials

Objectives:

- Factor expressions using grouping
- Factor over complex numbers
- Factor perfect cubes
- Factor perfect squares

Notes:

2.3 Assignment

Factor by grouping.

1. $4x^3 - 20x^2 + 4x - 20$
2. $x^3 + 3x^2 + 5x + 15$
3. $x^3 - 4x^2 + 5x - 20$
4. $x^6 - 3x^4 + x^2 - 3$

Factor the sum or difference of two cubes.

5. $y^3 - 8$
6. $z^3 + 64$
7. $8x^3y^3 - 64x^6$
8. $64z^3 + 27$

Factor the sum or difference of two squares.

9. $z^2 - 49$
10. $64 - 25y^2$
11. $x^2 + 9$
12. $16z^2 + 25$
13. $9y^2 - 16$
14. $36x^2 + 64$

Find the zeros of the function.

15. $x^3 + x^2 + 4x + 4$
16. $9x^2 - 3x - 2$
17. $5x^3 - 5x^2 - 10x$
18. $x^3 - 25x$
19. $x^6 - 9x^4 - x^2 + 9$
20. $4x^4 - 4x^2 - x^2 + 1$

2.4 Assignment! Mixed factoring…

Unit 2.5 Graphing Polynomial Functions

Objectives:

- Graph polynomial function using the roots of the function
- Analyze properties of polynomial functions

Notes:

Assignment 2.5

Identify the zeros for each of the following functions. State the multiplicity of each zero.

1. $y = (x-4)(x+5)^8(x-6)^3$
2. $y = 2x^7(x+8)^2(x-9)^3$
3. $y = -4x^5(x-1)(2x+5)^4$
4. $y = (7x+21)^9(x-4)^3(2x+10)$

Graph each polynomial function using a graphing calculator. Identify the x-intercepts, local max and min, intervals of increasing and decreasing, end behavior, and domain and range.

5. $y = (x-3)(x+6)$
6. $y = -2(x-2)(x-5)$
7. $y = (x+3)^2(x-1)$
8. $y = -4(x-2)(x+4)^3(x+6)$
9. $y = (x+1)^{24}(x-3)^2$
10. $y = -x(x+3)(x-4)$

Graph each polynomial function by hand. Identify the x-intercepts and end behavior.

11. $y = -6x(4x - 3)^2(2x + 8)^7$

12. $y = (6x + 12)(x - 3)(3x + 15)^3$

13. $y = 2x^4 + +8x^3 - 42x^2$

14. $y = -3x^2 - 12x^2 - 3x$

15. $y = x^3 + 3x^2 - 9x - 27$

16. $y = -x^5 - x^4 + 2x^3$

Unit 2.6 Dividing Polynomials Using Long Division and Synthetic Division

Student Learning Targets:

- I can divide polynomials using long division.
- I can find the remainder using the Remainder Theorem
- I can apply the ideas of the Factor Theorem
- I can divide polynomials using synthetic division.

Remainder Theorem:

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Factor Theorem:

A polynomial function $f(x)$ has a factor $x - k$, if and only if $f(k) = 0$.

Notes:

Assignment 2.6:

Simplify.

1. $\frac{4xy^2-2xy+2x^2y}{xy}$

2. $(3a^2b - 6ab + 5ab^2) \div (ab)$

Simplify using long division.

3. $(10x^2 + 15x + 20) \div (5x + 5)$

4. $\frac{x^4-3x^3+6x^2-3x+5}{x^2+1}$

5. $(18a^2 + 6a + 9) \div (3a - 2)$

6. $\frac{27y^2+27y-30}{9y-6}$

Use the Remainder Theorem to find the remainder when *f(x)* is divided by *x – k.*

7. $f(x) = 2x^2 - 3x + 1;\ k = 2$

8. $f(x) = x^4 - 5;\ k = 1$

Use the Factor Theorem to determine whether the first polynomial is a factor of the second polynomial.

9. $(x - 3);\quad x^3 - x^2 - x - 15$

10. $(x - 2);\quad x^3 + 3x - 4$

Simplify using synthetic division.

11. $(z^4 - 3z^3 + 2z^2 - 4z + 4) \div (z - 1)$

12. $\frac{y^3+11y^2-10y+6}{y+2}$

13. $\frac{2x^4-5x^3+7x^2-3x+1}{x-3}$

14. $(5x^4 - 3x + 1) \div (x - 4)$

15. $\frac{x^4-3x^2-18}{x-2}$

Unit 2.7: Extend Polynomial Identities to Complex Numbers

Student Learning Targets:

- Know what the Linear Factorization Theorem is and how to use it.
- Find complex zeros of a polynomial function.
- Write a linear factorization of a polynomial function.

Notes:

Assignment 2.7:

Write each polynomial in standard form.

1. $f(x) = (x + 1)(x - 3)(x + 5)$

2. $f(x) = (2x - 1)(x + 4i)(x - 4i)$

State the number of zeros each function has without graphing the function.

3. $f(x) = x^5 + 4x^3 - 6x^2 + 2x + 1$

4. $f(x) = -3x^7 + 8x^6 - 2x^3 + 4x^2$

Use the rational zeros theorem to identify all the possible rational zeros of each function.

5. $f(x) = x^4 + 8x + 32$

6. $f(x) = x^3 + x^2 + x - 28$

7. $f(x) = x^7 - 9x^5 + 5x^4 - 3x^2 + 7$

8. $f(x) = x^{12} + 12x^7 + 4x^3 + x - 30$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

9. $f(x) = x^3 - x^2 - 10x - 8;\ x + 2$

10. $f(x) = 2x^3 + 7x^2 - 53x - 28;\ x - 4$

11. $f(x) = 2x^3 + 17x^2 + 23x - 42;\ x - 1$

12. $f(x) = 6x^3 - 25x^2 + 2x + 8;\ 2x + 1$

Find all of the zeros of each function. Then write each polynomial in factored form.

13. $f(x) = x^3 + 10x^2 + 31x + 30$

14. $f(x) = x^4 - x^3 - x^2 - x - 2$

15. $f(x) = x^3 + 6x^2 - x - 30$

16. $f(x) = x^3 + x^2 + 4x + 4$

True or False. Justify your answer.

17. The polynomial $f(x) = 4x^3 + 7x + 5$ has three zeros.

18. The polynomial $f(x) = x^4 + 5x - 4$ has a zero at $x = 2$.

19. A polynomial with degree of 4 will cross the x-axis 4 times.