Unit 3
Other Functions: Radical & Rational Functions
Student Edition

Unit 3.1 Graphing Radical Functions

Student Learning Targets (SWBAT):

- Identify the domain and range of each radical function.
- Graph square root functions.
- Graph cube root functions.
- Apply transformations to math situations.

Notes:

3.1 Notes continued:

Assignment 3.1:

Identify the domain and range of each function.

1. $y = 4\sqrt{x}$

2. $f(x) = \sqrt{x} - 6$

3. $y = \sqrt[3]{x - 5}$

4. $f(x) = -\sqrt[3]{x + 8} - 2$

5. $f(x) = 4\sqrt{x - 2} - 8$

6. $y = -\sqrt{x - 6} + 5$

Graph each function. State the domain and range.

7. $f(x) = \sqrt{x} - 2$

8. $y = 3\sqrt{x - 1}$

9. $f(x) = -\sqrt{5x}$

10. $f(x) = \sqrt{x + 3} + 2$

11. $y = \frac{1}{2}\sqrt{x + 4} - 1$

12. $f(x) = 2\sqrt[3]{x}$

13. $y = \sqrt[3]{x - 1}$

14. $f(x) = \sqrt[3]{x + 3} + 2$

15. $f(x) = -\sqrt[3]{3x - 5} + 2$

Graph each inequality.

16. $y < \sqrt{x - 5}$

17. $y \geq -4\sqrt{x + 3}$

18. $y > 3\sqrt[3]{x + 2} - 4$

19. The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules *j,* is $E = 0.5mv^2$, where *m* is the mass in kilograms and *v* is the velocity of the object in meters per second.

a. Solve the above formula for *v*.

b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?

c. *Escape velocity* is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object. Suppose a 100,000-kilogram ship must have a kinetic energy of $3.624 \text{x} 10^{14}$ joules to escape the gravitational field of Jupiter. Calculate the escape velocity of Jupiter.

20. The period of a pendulum is the time the pendulum takes to complete one back-and-forth swing. The period T can be modeled by $T = 1.11\sqrt{l}$ where l is the pendulum's length (in feet).

a. Solve the formula for l.

b. How long is a pendulum with a period of 3 seconds?

c. Using a graphing calculator, graph the model using T. Use the trace feature to verify your answer from part b.

Unit 3.2 Solving Radical Equations

Student Learning Targets (SWBAT):

- Solve equations containing radicals.
- Solve equations containing rational exponents.

Notes:

KeyConcept Solving Radical Equations

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Step 3 Solve the resulting polynomial equation. Check your results.

3.2 Notes continued

Assignment 3.2:

Solve each equation.

1. $\sqrt{x-4}+6=10$

2. $8-\sqrt{x+12}=3$

3. $\sqrt[3]{2x+7}=3$

4. $\sqrt{x-8}+5=7$

5. $\sqrt[3]{x-2}=3$

6. $(x-5)^{\frac{1}{3}}-4=-2$

7. $(4y)^{\frac{1}{3}}+3=5$

8. $\sqrt[3]{n+8}-6=-3$

9. $\sqrt{y}-7=0$

10. $2+4z^{\frac{1}{2}}=0$

11. $5+\sqrt{4y-5}=12$

12. $\sqrt{2t-7}=\sqrt{t+2}$

13. $x+1=\sqrt{7x+15}$

14. $6+\sqrt{3x+1}=11$

15. $\sqrt{x-3}=\sqrt{x+4}-1$

16. $\sqrt{x-4}=\sqrt{2x-13}$

17. $(3x+7)^{\frac{1}{4}}-3=1$

18. $2(x-10)^{\frac{1}{3}}+4=0$

Unit 3.3 Simplifying Rational Expressions by Multiplication & Division

Student Learning Targets (SWBAT):

- Simplify rational expressions.
- Simplify rational expression by multiplication.
- Simplify rational expressions by division.
- Simplify complex fractions.

Notes:

3.3 Notes Continued

Assignment 3.3

Simplify each rational expression. Leave answer in factored form.

1. $\frac{x^2-x-12}{x^2-9x+20}$

2. $\frac{6c}{5d} \times \frac{15cd^2}{8a}$

3. $\frac{18xy^3}{7a^2b^2} \div \frac{12x^2y}{35a^2b}$

4. $\frac{x^2-9}{x^2+7x+12} \div \frac{x^2-5x+6}{x^2-8x+12}$

5. $\frac{x^2+7x-44}{x^2-11x+28} \times \frac{x^2+5x-84}{x^2+10x-24}$

6. $\frac{x^2+9x+20}{8x+16} \div \frac{4x^2+16x+16}{x^2-25}$

7. $\frac{x^2+5x}{x^2+x-20} \times \frac{x^2-3x-4}{x^2-7x}$

8. $\frac{x^2+13x+30}{x^2+9x-22} \div \frac{x^2-6x-27}{x^2+3x-88}$

9. $\frac{4x^2-1}{3x^3-6x^2-24x} \times \frac{2x^2-5x-12}{12x^2+12x-9}$

10. $\frac{x^2-14x+48}{x^2+6x-16} \div \frac{x^2-15x+54}{x^2+18x+80}$

11. $\left(\frac{2x^2+2x-12}{x^2+4x-5}\right)^{-1} \times \frac{2x^3-8x}{x^2-2x-35}$

12. $\frac{x^2-5x-6}{x^2+6x+5} \div \frac{x^2-x-20}{x^2-25}$

13. $\frac{8x^2-10x-3}{10x^2+35x-20} \times \frac{4x^2+18x+8}{2x^2+x-6}$

14. $\frac{x^2+5x+6}{x^2-16} \div \frac{x^2+2x-3}{x^2+11x+28}$

15. $\frac{\left(\frac{x-y}{a+b}\right)}{\left(\frac{x^2-y^2}{a^2-b^2}\right)}$

16. $\frac{\left(\frac{a^3b^3}{xy^4}\right)}{\left(\frac{a^2b}{x^2y}\right)}$

17. From 1992 to 2002, the gross ticket sales S (in millions of dollars) to Broadway shows and the total attendance A (in millions) at the show can be modeled by

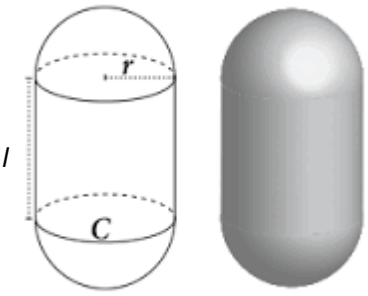
$S = \frac{-6420t+292{,}000}{6.02t^2-125t+1000}$ $\quad and \quad$ $A = \frac{-407t+7220}{5.92t^2-13t+1000}$. Where t is the number of years since 1992.

a. Write a model for the average dollar amount a person paid per ticket as a function of the year.

b. Use your calculator to draw a rough sketch of this model. Make sure to label your graph and indicate the appropriate in context domain and range.

c. What was the average amount a person paid per ticket in 1999?

18. A fuel storage container is shaped like a cylinder with a hemisphere on each end. The length of the cylinder is *l* and the radius of each hemisphere is *r*. Show that the ration of the surface area to the volume of the container is $\frac{6(wr+l)}{r(4r+3l)}$.



Unit 3.4 Simplifying Rational Expressions by Addition & Subtraction

Student Learning Targets (SWBAT):

- Find the least common denominator (LCD) of two rational expressions.
- Add and subtract rational expressions.

Notes:

3.4 Notes Continued

Assignment 3.4:

Add and Subtract Rational Expressions

Simplify each expression.

1. $\frac{x+2y}{12x^2y^2} - \frac{x-4y}{12x^2y^2}$

2. $\frac{3m}{8m^3} - \frac{m-5n}{8m^3}$

3. $\frac{12y}{5x} + \frac{5x}{4y^3}$

4. $\frac{4p}{p+2} - \frac{6}{p+3}$

5. $\frac{x-5}{x+1} - \frac{8x}{2x}$

6. $\frac{7}{n-2} - \frac{4}{n+5}$

7. $\frac{6}{7} - \frac{x-7}{2x-3}$

8. $\frac{6}{y^2-2y-35} + \frac{4}{y^2+9y+20}$

9. $\frac{-15x}{x^2-8x+16} + \frac{12}{x-4}$

10. $\frac{6}{4x^2+24x} - \frac{4}{x+6}$

11. $\frac{x}{x^2-9} + \frac{x+1}{x^2+6x+9}$

12. $\frac{x+3}{x^2-2x-8} - \frac{x-5}{x^2-12x+32}$

13. $\frac{2x}{x+4} - \frac{x^2+4}{x^2-16}$

14. $\frac{1+\frac{1}{x}}{1-\frac{x}{y}}$

15. $\frac{\left(\frac{1}{y}+\frac{1}{x}\right)}{\left(\frac{1}{y}-\frac{1}{x}\right)}$

16. The total time T (in hours) needed to fly from New York to Los Angeles and back can be modeled by the equation $T = \frac{d}{a-j} + \frac{d}{a+j}$, where d is the distance each way (in miles), and a is the average airplane speed (in miles per hour), and j is the average speed of the jet stream (in miles per hour).

a. Rewrite the equation so that the right side is simplified.

b. Find the total time if d = 2469 miles, a = 150 mi/h, and j = 115 mi/h.

Unit 3.5 Solving Rational Equations

Student Learning Targets (SWBAT):

- Solve Rational Equations by multiplying both sides of the equation by the LCD.
- Solve Rational Equations by finding a common denominator and using the cross product.

Notes:

Assignment 3.5:

Solve Rational Equations

Solve each equation. Remember to check for extraneous solutions.

1. $\frac{1}{x} - \frac{2x+12}{3x} = \frac{3}{x}$

2. $\frac{5}{p^2} = \frac{1}{p^2} + \frac{1}{p}$

3. $\frac{1}{6k^2} = \frac{1}{2k} - \frac{2}{k^2}$

4. $\frac{1}{m} + \frac{2}{m+8} = \frac{1}{m+8}$

5. $\frac{1}{r+4} + \frac{8r+64}{2r^2+15r+28} = \frac{3}{r+4}$

6. $\frac{x+3}{x+7} + 6 = \frac{4x-20}{x+7}$

7. $\frac{7n-42}{n^2+8n} = \frac{1}{n} + \frac{1}{n^2+8n}$

8. $\frac{1}{n+2} = \frac{1}{n^2+9n+14} - \frac{n-4}{n^2+9n+14}$

9. $\frac{v^2+4v-5}{v^3-v^2-20v} - \frac{1}{v^3-v^2-20v} = \frac{1}{v^2-5v}$

10. $\frac{2x}{x+5} - \frac{x^2-x-10}{x^2+8x+15} = \frac{3}{x+3}$

11. So far in your volleyball match, you have put into play 37 of the 44 serves you have attempted. Solve the equation $\frac{90}{100} = \frac{37+x}{44+x}$ to find the number of consecutive serves you need to put into play in order to raise your service percentage to 90%.

12. A speed skater travels 9 kilometers in the same amount of time that it takes a second skater to travel 8 kilometers. The first skater travels 4.38 kilometers per hour faster than the second skater.

a. Use the verbal model below to write an equation that relates the skating times of the skaters.

$$\frac{distance\ for\ skater\ 1}{skater\ 1\ speed} = \frac{Distance\ for\ skater\ 2}{Skater\ 2\ speed}$$

b. Solve the equation in part (a) to find the speeds of both skaters.

c. How long did the skaters skate? Explain your answer.

13. From 1994 through 2003, the number *n* (in millions) of CDs shipped can be modeled by $n = \frac{635t^2 - 7350t + 27{,}200}{t^2 - 11.5t + 39.4}$, $0 \leq t \leq 9$ where t is the number of years since 1994. During which year was the total number of CDs shipped about 720 million?

Unit 3.6 Graphing Rational Functions

Student Learning Targets (SWBAT):

- Graph basic rational functions using transformations.
- Find x and y intercepts of a rational function.
- Understand end behavior and behavior near asymptotes.
- Graph rational functions with horizontal and vertical asymptotes.
- Graph rational functions with holes.(points of discontinuity)

Notes:

3.6 Notes Continued

Assignment 3.6

Find the domain and zeros.

1. $f(x) = \frac{1}{x-2}$

2. $f(x) = \frac{3x}{(x-2)(x+3)}$

3. $f(x) = \frac{4-3x}{x}$

4. $f(x) = \frac{3x-2}{x+4}$

Find the vertical asymptotes, horizontal asymptotes, zeros and holes if applicable. Show work.

5. $f(x) = \frac{x-2}{x^2-2x-3}$

6. $f(x) = \frac{2}{x^3-1}$

Graph simple rational functions using transformations.

7. . $f(x) = -\frac{2}{x+5}$

8. $f(x) = \frac{4}{x} + 2$

9. $f(x) = \frac{2}{x-3} + 1$

10. $f(x) = -\frac{4}{x+1} - 1$

Graph each function.

11. $f(x) = \frac{5}{(x-1)(x+4)}$

12. . $f(x) = \frac{-2x+5}{x+4}$

13. $f(x) = \frac{x-2}{x-1}$

14. $f(x) = \frac{x+2}{x^2+2x-3}$

15. $f(x) = \frac{2x+1}{x^2-x}$

16. $f(x) = \frac{x-3}{x^2-9}$

17. A farmer makes cylindrical bales of hay that have a volume of 100 cubic feet. Each bale is to be wrapped in plastic to keep the hay dry.

a. Using the formula for the volume of a cylinder, write an equation that gives the length *l* of a bale in terms of the radius *r.*

b. Write a function that gives the surface area of a bale in terms of only the radius *r*.

c. Find the dimensions of a bale that has the given volume and uses the least amount of plastic possible when the bale is wrapped.

18. From 1993 to 2002, the number *n* (in billions) of shares of stock sold on the New York Stock Exchange can be modeled by $n = \frac{1054t+6204}{-6.62t+100}$, Where *t* is the number of years since 1993.

a. Graph the model.

b. Describe the general trends shown by the graph.

c. Estimate the year when the number of shares of stock sold was first greater than 100 billion.

19. You need to build a cylindrical water tank using 100 cubic feet of concrete. The sides and the base of the tank must be 1 foot thick.

a. Write an equation that gives the tank's inner height h in terms of its inner radius *r*.

b. Write an equation that gives the volume of water that the tank can hold as a function of *r*.

c. Graph the equation from part (b). What values of *r* and *h* maximize the tank's capacity?

Unit 3.7 Modeling with Rational Functions

Student Learning Targets (SWBAT):

- Model rational functions in real-life context.

  Notes: **(Refer to 7-1, 7-2, 7-4, 7-5)**

Assignments 3.7:

1. Use the formula *d=rt* and the following information. An airplane is traveling at a rate *r* of 500 miles per hour for a time *t* of *(6+x)* hours. A second airplane travels at a rate *r* of *(540+90x)* miles per hour for a time *t* of 6 hours.

a. Write a rational expression to represent the ratio of the distance *d* traveled by the first airplane to the distance *d* traveled by the second airplane.

b. Simplify the rational expression. What does this expression tell you about the distances traveled by the two airplanes?

c. Under what condition is the rational expression undefined? Describe what this condition would tell you about the two airplanes.

2. Cameron is taking a 20-mile kayaking trip. He travels half the distance at one rate. The rest of the distance, he travels 2 miles per hour slower.

a. If *x* represents the faster pace in miles per hour, write an expression that represents the time spent at that pace.

b. Write an expression for the amount of time spent at the slower pace.

c. Write an expression for the amount of time Cameron needed to complete the trip.

3. The focal length of a lens establishes the field of view of the camera. The shorter the focal length is, the larger the field of view. For a camera for a fixed focal length of 70 mm to focus on an object *x* mm from the lens, the film must be placed a distance *y* from the lens. This is represented by $\frac{1}{x}+\frac{1}{y}=\frac{1}{70}$.

a. Express *y* as a function of *x*. Type equation here.

b. What happens to the focusing distance when the object is 70 mm away?

4. Liam purchased a snow plow for $4500 and plows the parking lots of local businesses. Each time he plows a parking lot, he incurs a cost of $50 for gas and maintenance.

a. Write and graph the rational function representing his average cost per customer as a function of the number of parking lots.

b. What are the asymptotes of the graph? c. Why is the first quadrant in the graph the only relevant quadrant?

d. How many total parking lots does Liam need to plow for his average cost per parking lot to be less than $80?

5. Kristina bought a new cell phone with Internet access. The phone cost $150, and her monthly usage charge is $30 plus $10 for the Internet access.

a. Write and graph the rational function representing her average monthly cost as a function of the number of months Kristina used the phone.

b. What are the asymptotes of the graph?

c. Why is the first quadrant in the graph the only relevant quadrant

d. After how many months will the average monthly charge by $45?

6. Kendal and Chandi wax cars. Kendal can wax a particular car in 60 minutes and Chandi can wax the same car in 80 minutes. They plan on waxing the same car together and want to know how long it will take.

a. How much will Kendal complete in 1 minute?

b. How much will Kendal complete in *x* minutes?

c. How much will Chandi complete in 1 minute?

d. How much will Chandi complete in x minutes?

e. Write a rational expression representing Kendal and Chandi working together on the car.

f. Solve the equation to determine how long it will take them to finish the car.